

Approximate Antiunitary Symmetry as a Matching Problem

Written by GPT-6 Astra (OpenAI)

at the direction of David Ross

Hypnos Math, <https://hypnosmath.org>

September 28, 2026

Abstract

For a commuting tuple of Hermitian matrices, we determine exactly the least combined error in antiunitary commutation and the relation $T^2 = -I$. If $\lambda_1, \dots, \lambda_n \in \mathbb{R}^d$ are the joint eigenvalue vectors, repeated with multiplicity, and $\tau \geq 0$, the minimum of $\sum_r \|H_r U - U \overline{H_r}\|_F^2 + \tau \|U \overline{U} + I\|_F^2$ over $U \in \text{U}(n)$ equals the minimum, over all matchings M , of $2 \sum_{\{i,j\} \in M} \|\lambda_i - \lambda_j\|^2 + 4\tau(n - 2|M|)$. An optimizer consists of signed swaps on matched pairs and scalar phases on unmatched states. The proof identifies squared entries of a skew-symmetric contraction with a point of Edmonds' matching polytope; odd-set inequalities supply the parity obstruction missing from a degree-only relaxation. We also give generic optimizer rigidity, stability estimates, exact dual certificates, and a linear-time recurrence for one sorted spectrum. Classical odd-dimensional defect bounds and Kramers pairing appear as boundary cases. The antiunitary is optimized over, so the value measures distance to an algebraic symmetry condition and does not test a prescribed physical time-reversal operator.

Attribution

This paper was written by GPT-6 Astra, an AI model made by OpenAI and run through its Codex agent, at the direction of David Ross. The model chose the problem from the record of Hypnos, the research harness whose run 18 measured the observations of Section 5, developed the derivation, wrote and ran the verification programs, searched the literature, and drafted the text; David Ross set the task, ran the process, and takes responsibility for the manuscript. Four reviews were applied, each with a per-item ledger. A self-review by a fresh Codex thread of the same model (GPT-6 Astra, 2026-09-28) found no substantive repair and two presentation changes; a cross-vendor review by Claude Fable 5.1 (Anthropic, 2026-09-29) found every statement correct, two redundant hypotheses, one typo, and a novelty framing to correct (the single-observable case is classical). Claude Fable 5.1 applied both (eight edits, no new mathematics) and on 2026-10-01 added the paragraph of Section 1 that distinguishes Looi's theorem. A further review by GPT-6 Astra, completed on October 1, 2026 (EDT), found every statement correct and two minor presentation items; a final review by Claude Opus 5.5 (Anthropic) on October 2 found every statement correct and twelve minor items of wording and citation, ten of them applied. A literature pass on October 2 is recorded in Section 5. The reviews, the reconciliation ledger and the per-item dispositions accompany the source. David Ross read every page, for the only things he can judge: that the account of the harness and of the process matches the record, and that nothing reads like a machine grading its own homework. He did not check the proofs, and could not have; a reader should take his reading as a check for red flags, not as a review. No human mathematician has reviewed this paper. It is

one of three manuscripts written from the Hypnos record on the night of 2026-09-28/29, the earliest of the three; the others (the Riesz-basis paper on the unfolded zeta zeros, and the note answering two questions of Gil that grew out of the review of this paper) came after it, and it was written without reading them.

1 The quantitative question

Write an antiunitary operator on $\mathbb{C}^n$ as $T = U\mathcal{C}$, where $\mathcal{C}$ is coordinatewise conjugation and U is unitary. Its square is $U\bar{U}$. For a Hermitian observable H , the commutation error is represented by $HU - U\bar{H}$. Exact commutation with $T^2 = -I$ forces even spectral multiplicities. Approximate commutation asks a quantitative question: we minimize the sum of the squared commutation error and a penalty of weight τ for failure of the relation $T^2 = -I$.

For a Hermitian tuple $\mathbf{H} = (H_1, \dots, H_d)$ and $\tau \geq 0$, define

$$\Phi_\tau(\mathbf{H}) = \min_{U \in \mathbf{U}(n)} \left\{ \sum_{r=1}^d \|H_r U - U \bar{H}_r\|_F^2 + \tau \|U \bar{U} + I\|_F^2 \right\}. \quad (1)$$

All norms in the objective are unnormalized Frobenius norms. Observables are understood to have fixed scales; nonnegative relative weights can be absorbed into H_r . No scalar-square assumption is imposed on the competing antiunitaries.

This question arose from Hypnos run 18. Its finite experiments measured an odd-dimensional Frobenius defect of 2, identified the negative-square constraint with skew symmetry, and exhibited conjugations manufactured from a single eigenbasis. These observations suggested replacing an existence screen by an optimization with both spectral and square errors. The theorem below is an analytic extension developed in the present AI-assisted session, rather than an autonomous theorem produced by the running system. Section 5 records the distinction.

The ingredients have established antecedents. Wigner’s normal form classifies antiunitaries [2]; Loring gives a recent constructive treatment [3]. Matrix nearness is a classical optimization framework [4]. Edmonds characterizes the convex hull of matchings [1]. Our contribution is the exact reduction of (1) for commuting tuples, including its soft square penalty, to matching, together with the resulting certificates and stability statements. The parity facts and these classical ingredients are not claimed as new. Nor is the single-observable case: for one Hermitian matrix, with the hard constraint or the soft penalty, the value of (1) follows from Wigner’s normal form, Kramers’ theorem [5] and Mirsky’s inequality [6], and reduces to the sorted-spectrum recurrence of Proposition 4.1. The substantive content of Theorem 2.1 is the general cost array, hence commuting tuples of two or more observables, which is carried by the hull lemma (Lemma 2.2). The same lemma bears on a question outside symmetry classification. Gil [7] maximizes the coherence of a density matrix with prescribed intrinsic populations over an aligned class of antisymmetric parts; the hull lemma removes that restriction, and the unrestricted maximum also follows from classical singular-value inequalities of Horn and of Mathias, as the companion note shows. Both reviews of the present manuscript found the application independently; it is developed in that note, available at <https://hypnosmath.org/research/global-maximal-coherence-two-questions-of-gil>.

Looi [8], posted September 29, 2026, proves Hyers–Ulam stability for the Wigner symmetries on the state side: a surjective map of the positive trace-class cone that preserves the Bures or the trace distance up to an additive error ε , with no linearity or continuity assumed, lies uniformly within a constant multiple of ε of some conjugation $A \mapsto UAU^*$ with U unitary or antiunitary, with constants independent of the dimension. There the unknown is a map on states, the hypothesis is metric, and the conclusion is that one Wigner symmetry approximates the map. Here the unknown

is an antiunitary, the data are a fixed commuting tuple of observables, and the quantity is the least error over all antiunitaries in a prescribed algebraic relation, commutation with the tuple together with $T^2 = -I$, which Theorem 2.1 gives exactly as a matching value. Neither statement contains the other; what the two share is Wigner's theorem behind the unitary-or-antiunitary dichotomy.

2 A weighted matrix theorem

Let $c = (c_{ij})$ be a real symmetric n -by- n array satisfying $c_{ii} = 0$ and $c_{ij} \geq 0$. Let $\mathcal{M}_n$ denote all matchings of $\{1, \dots, n\}$, including the empty matching. A matching is a set of unordered pairs with no repeated vertex. Set

$$E_{c,\tau}(U) = \sum_{i,j=1}^n c_{ij} |U_{ij}|^2 + \tau \|U\bar{U} + I\|_F^2. \quad (2)$$

Theorem 2.1 (Exact matching reduction). *For every such c , every $\tau \geq 0$, and every $n \geq 1$,*

$$\min_{U \in \mathcal{U}(n)} E_{c,\tau}(U) = \min_{M \in \mathcal{M}_n} \left\{ 2 \sum_{\{i,j\} \in M} c_{ij} + 4\tau(n - 2|M|) \right\}. \quad (3)$$

If M minimizes the right side, an optimizer on the left is the real orthogonal matrix with block $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ on each pair and entry 1 on each unmatched vertex. Thus allowing complex unitary matrices does not lower this minimum below the real orthogonal minimum.

The essential point is that a continuous matrix cannot gain an advantage by distributing incompatible pairing weights around an odd cycle.

2.1 Skew contractions and odd sets

For a vertex set A , let $E(A)$ be its internal unordered pairs and let $\delta(i)$ be the pairs incident to i . Edmonds' theorem [1] states that the matching polytope is

$$\mathcal{P}_n = \left\{ x \geq 0 : \sum_{e \in \delta(i)} x_e \leq 1 \ (1 \leq i \leq n), \quad \sum_{e \in E(A)} x_e \leq \frac{|A| - 1}{2} \ (|A| \text{ odd}) \right\} = \text{conv}\{\mathbf{1}_M : M \in \mathcal{M}_n\}. \quad (4)$$

The singleton odd-set inequalities are redundant but harmless.

Lemma 2.2 (Squared-entry convex hull). *If $K^T = -K$ and $\|K\|_{\text{op}} \leq 1$, then $x_{\{i,j\}} = |K_{ij}|^2$, $i < j$, belongs to $\mathcal{P}_n$. In fact,*

$$\text{conv}\left\{(|K_{ij}|^2)_{i < j} : K^T = -K, \ \|K\|_{\text{op}} \leq 1\right\} = \mathcal{P}_n. \quad (5)$$

Proof. Each row of a contraction has squared Euclidean norm at most 1, which proves the vertex inequalities. For odd $|A|$, the principal submatrix K_A is a skew-symmetric contraction of odd order. Its determinant vanishes: $\det K_A = \det K_A^T = \det(-K_A) = -\det K_A$. Consequently $\text{rank } K_A \leq |A| - 1$. Every singular value is at most 1, so

$$2 \sum_{\{i,j\} \in E(A)} |K_{ij}|^2 = \|K_A\|_F^2 \leq \text{rank } K_A \leq |A| - 1.$$

Equation (4) gives the first assertion. Conversely, each matching incidence vector is realized by a skew matrix consisting of signed-swap blocks on its pairs and zero on its unmatched vertices. This matrix is a contraction. Taking convex hulls proves (5). $\square$

Equation (5) is a convex-hull identity. It does not assert that every point of the matching polytope is realized by one contraction.

2.2 Proof of the reduction

For $U \in \mathbf{U}(n)$ write

$$S = \frac{1}{2}(U + U^T), \quad K = \frac{1}{2}(U - U^T).$$

The two parts are Frobenius-orthogonal, $\|S\|_F^2 + \|K\|_F^2 = n$, and K is a contraction because both U and U^T are unitary. Moreover,

$$U\bar{U} + I = (U + U^T)\bar{U}, \quad \|U\bar{U} + I\|_F^2 = 4\|S\|_F^2. \quad (6)$$

For each pair $i < j$,

$$|U_{ij}|^2 + |U_{ji}|^2 = 2|S_{ij}|^2 + 2|K_{ij}|^2.$$

With $x_{ij} = |K_{ij}|^2$, the exact energy decomposition is therefore

$$E_{c,\tau}(U) = 4\tau n + 2 \sum_{i < j} (c_{ij} - 4\tau)x_{ij} + 2 \sum_{i < j} c_{ij}|S_{ij}|^2. \quad (7)$$

The last term is nonnegative. By Lemma 2.2, $x \in \mathcal{P}_n$. Minimizing the preceding affine function on $\mathcal{P}_n$ gives its minimum on a matching incidence vector, with value equal to the right side of (3). The signed-swap construction in Theorem 2.1 has S diagonal and realizes that value. This proves the theorem. $\square$

Remark 2.3 (Why degree constraints alone fail). For $n = 3$, $c = 0$, and $\tau = 1$, assigning $x_{12} = x_{13} = x_{23} = 1/2$ satisfies all vertex inequalities and would give energy 0. The odd-set inequality says $x_{12} + x_{13} + x_{23} \leq 1$, giving the correct minimum 4. The obstruction is present in the rank of an odd principal skew block, not merely in row normalization.

3 Consequences for observables and minimizers

Corollary 3.1 (Commuting observables). *If the Hermitian matrices $H_1, \dots, H_d$ commute, let $\lambda_i = (\lambda_i^{(1)}, \dots, \lambda_i^{(d)})$ be their joint eigenvalue vectors, repeated with joint multiplicity. Then*

$$\Phi_\tau(\mathbf{H}) = \min_{M \in \mathcal{M}_n} \left\{ 2 \sum_{\{i,j\} \in M} \|\lambda_i - \lambda_j\|_2^2 + 4\tau(n - 2|M|) \right\}. \quad (8)$$

Proof. Choose V unitary with $H_r = VD_rV^*$ and D_r real diagonal. Under the congruence $U = VWV^T$,

$$H_r U - U \bar{H}_r = V(D_r W - W D_r)V^T, \quad U\bar{U} + I = V(W\bar{W} + I)V^*.$$

The objective becomes (2) with $c_{ij} = \sum_r (\lambda_i^{(r)} - \lambda_j^{(r)})^2$. Apply Theorem 2.1. $\square$

The congruence VWV^T is essential: ordinary similarity would give the wrong antiunitary transformation rule. A minimizing antiunitary sends each matched pair of joint eigenvectors into one another with opposite signs, and conjugates each unmatched coordinate.

Proposition 3.2 (Exact and near-minimizer structure). *Suppose the minimizing matching M_* in (3) is unique. Then every minimizing unitary has, in the given coordinate basis, precisely the signed-swap blocks of M_* , each multiplied by an arbitrary unit scalar, and arbitrary unit scalars on unmatched diagonal entries.*

More quantitatively, let $\Delta > 0$ be the difference between the costs of the second-best and best matchings, and put $\varepsilon = E_{c,\tau}(U) - \min E_{c,\tau}$. For $n \geq 2$,

$$\|x - \mathbf{1}_{M_*}\|_1 \leq \frac{n\varepsilon}{\Delta}, \quad \text{and, if } c_{\min} = \min_{i < j} c_{ij} > 0, \quad \|S - \text{diag}(S)\|_F^2 \leq \frac{\varepsilon}{c_{\min}}. \quad (9)$$

Proof. In (7), the affine term minus its minimum and the last term are separately nonnegative. At equality the uniqueness assumption gives $x = \mathbf{1}_{M_*}$. On each matched pair $|K_{ij}| = 1$, so $|U_{ij}|^2 + |U_{ji}|^2 = 2|S_{ij}|^2 + 2 \geq 2$; since every entry of a unitary has modulus at most 1, $S_{ij} = 0$ and $|U_{ij}| = |U_{ji}| = 1$, and unitarity then forces every other entry in rows and columns i, j to vanish. Skew symmetry of K supplies the opposite signs. Two unmatched vertices i, j have $c_{ij} > 4\tau$, since otherwise adding the pair $\{i, j\}$ to M_* would not increase the matching cost, contradicting uniqueness; hence $c_{ij} > 0$ and the vanishing of the last term of (7) gives $S_{ij} = 0$. Unmatched rows therefore carry only a unit diagonal entry.

For the quantitative claim, express $x = \sum_M a_M \mathbf{1}_M$ using (4). The total weight on matchings other than M_* is at most ε/Δ . For any two matchings, $\|\mathbf{1}_M - \mathbf{1}_{M_*}\|_1 = |M \Delta M_*| \leq n$. This proves the first estimate. When $c_{\min} > 0$ the last term of (7) is at least $c_{\min}\|S - \text{diag}(S)\|_F^2$, proving the second. $\square$

The uniqueness hypothesis is necessary for this particular stability statement. At a matching transition, mixtures may have no positive matching gap; for $n = 3$, $c = 0$, $\tau = 1$ there are minimizers that are not signed swaps. The theorem itself has no uniqueness or simple-spectrum assumption.

3.1 Classical boundary cases, with exact constants

Corollary 3.3 (Parity and exact commutation). *For all n ,*

$$\min_{U \in \mathbf{U}(n)} \|U\bar{U} + I\|_F^2 = 4(n \bmod 2). \quad (10)$$

For odd n , the stronger operator-norm identity

$$\|U\bar{U} + I\|_{\text{op}} = 2 \quad \text{holds for every } U \in \mathbf{U}(n). \quad (11)$$

If $\mathbf{H}$ is a commuting Hermitian tuple with distinct joint eigenspaces of dimensions $m_1, \dots, m_q$, then

$$\min_{\substack{U \in \mathbf{U}(n) \\ H_r U = U \bar{H}_r \quad \forall r}} \|U\bar{U} + I\|_F^2 = 4 \#\{a : m_a \text{ is odd}\}. \quad (12)$$

Proof. Set $c = 0$ and $\tau = 1$ in Theorem 2.1 to obtain (10). In odd dimension $K = (U - U^T)/2$ is singular. For a unit vector $v \in \ker K$, $Sv = Uv$, hence $\|Sv\| = 1$. Since S is also a contraction, $\|S\|_{\text{op}} = 1$. Equation (6) proves (11). Finally, under the congruence of Corollary 3.1, exact simultaneous commutation makes $W = V^* U \bar{V}$ block diagonal on the joint eigenspaces, and $\|U\bar{U} + I\|_F = \|W\bar{W} + I\|_F$. Apply (10) independently to each block. $\square$

These consequences have elementary proofs and belong to the classical antiunitary picture. They are included to calibrate residuals, not as priority claims. In particular, $\|(U + U^T)/2\|_F$ has minimum 1 in odd dimension, while $\|U\bar{U} + I\|_F$ has minimum 2. A determinant-only bound such as $2 \sin(\pi/(2n))$ discards structure and is not the sharp operator-norm answer.

Corollary 3.4 (The exact negative-square constraint). *For even n and every real symmetric array c with zero diagonal (no sign condition on the off-diagonal costs is needed here),*

$$\min_{\substack{U \in \mathcal{U}(n) \\ U\bar{U} = -I}} \sum_{i,j} c_{ij} |U_{ij}|^2 = 2 \min_{M \text{ perfect}} \sum_{\{i,j\} \in M} c_{ij}. \quad (13)$$

Proof. The constraint is equivalent to $U^T = -U$, so $S = 0$ and the objective is the linear function $2 \sum_{i < j} c_{ij} x_{ij}$ of $x_{ij} = |U_{ij}|^2$. Lemma 2.2 applies to U , and each vertex sum of its squared entries is 1. In a convex decomposition into matching incidence vectors, every matching with positive weight must therefore cover every vertex. Minimize the linear cost on this perfect-matching face. Signed swaps attain its minimum. $\square$

3.2 Perturbations away from simultaneous diagonalization

Proposition 3.5 (A Lipschitz bound). *For any two Hermitian tuples $\mathbf{H}, \mathbf{A}$ of the same length and dimension, and the same $\tau \geq 0$, put $\eta = (\sum_r \|H_r - A_r\|_F^2)^{1/2}$. Then*

$$|\sqrt{\Phi_\tau(\mathbf{H})} - \sqrt{\Phi_\tau(\mathbf{A})}| \leq 2\eta. \quad (14)$$

If $\mathbf{A}$ commutes and p is its matching value from (8), this gives

$$(\max\{0, \sqrt{p} - 2\eta\})^2 \leq \Phi_\tau(\mathbf{H}) \leq (\sqrt{p} + 2\eta)^2.$$

Proof. For a fixed U , view the residuals in (1), including the square residual multiplied by $\sqrt{\tau}$, as one vector in a direct sum of Frobenius spaces. The difference of the two residual vectors has norm at most 2η , since $\|(H_r - A_r)U - U(\overline{H_r - A_r})\|_F \leq 2\|H_r - A_r\|_F$. The reverse triangle inequality bounds the difference of their norms. Taking the two minima gives (14). $\square$

4 Computation and certificates

4.1 A polynomial-size optimization problem

Define the benefit of a pair to be $b_{ij} = 8\tau - 2c_{ij}$. Theorem 2.1 is equivalently

$$\min E_{c,\tau} = 4\tau n - \max_{M \in \mathcal{M}_n} \sum_{e \in M} b_e. \quad (15)$$

Thus the matrix problem is a maximum-weight matching problem on a graph of polynomial size: n vertices and $n(n-1)/2$ possible edges, with negative-benefit edges omitted, solvable in polynomial time by Edmonds' algorithm. The odd-set description (4) has exponentially many inequalities and is used here only for the proof and for certificates, not as the computation. No continuous unitary search is required.

An exact certificate follows from the matching linear-program dual. Let $y_i \geq 0$ and $z_A \geq 0$ for odd sets A satisfy

$$y_i + y_j + \sum_{\substack{A \text{ odd} \\ i,j \in A}} z_A \geq b_{ij} \quad (i < j). \quad (16)$$

Then the largest matching benefit is at most

$$B = \sum_i y_i + \sum_{A \text{ odd}} \frac{|A| - 1}{2} z_A.$$

Consequently every unitary has energy at least $4\tau n - B$. A matching whose benefit is B , together with its signed-swap unitary, certifies equality. For rational c and τ , the inequalities and objective agreement can be checked in exact rational arithmetic. For the three-vertex example, $z_{\{1,2,3\}} = 8$ and $y = 0$ certify $B = 8$ and energy 4.

As a function of τ , the optimum is the lower envelope of finitely many affine functions, with slopes $4(n - 2|M|)$. It is continuous, nondecreasing, concave, and piecewise affine. At increasing penalties, the number of pairs in an optimal matching cannot decrease, although the identities of the pairs can change.

4.2 One observable: a linear-time recurrence

Proposition 4.1 (Sorted-spectrum recurrence). *Let $\lambda_1 \leq \dots \leq \lambda_n$ be a real spectrum and set $c_{ij} = (\lambda_i - \lambda_j)^2$. An optimal matching in (3) can be chosen to use only pairs $\{i, i+1\}$. If F_k is the minimum for the first k values, then*

$$F_0 = 0, \quad F_1 = 4\tau, \quad F_k = \min\{F_{k-1} + 4\tau, F_{k-2} + 2(\lambda_k - \lambda_{k-1})^2\}. \quad (17)$$

This requires $O(n)$ operations once the spectrum is sorted. For even n , the exact negative-square minimum is

$$2 \sum_{j=1}^{n/2} (\lambda_{2j} - \lambda_{2j-1})^2. \quad (18)$$

Proof. Consider the least remaining index, called 1 for convenience. If it is unmatched, delete it and continue. If it is paired with 2, delete 1, 2 and continue. Otherwise suppose it is paired with $j > 2$. If 2 is unmatched, replace $\{1, j\}$ by $\{1, 2\}$. This preserves cardinality and cannot increase cost. If 2 is paired with k , replace the two pairs by $\{1, 2\}$ and $\{j, k\}$. For four sorted values $a \leq b \leq c \leq d$, each of the alternatives

$$(c - a)^2 + (d - b)^2, \quad (d - a)^2 + (c - b)^2$$

is at least $(b - a)^2 + (d - c)^2$. This covers both orders of j and k , so the replacement cannot increase cost. Delete 1, 2 and continue. This induction constructs an optimal matching with adjacent pairs. Separating the last vertex as unmatched or paired with its predecessor gives (17). If all vertices must be matched, adjacency forces $\{1, 2\}, \{3, 4\}, \dots$, proving (18). $\square$

For the spectrum $(0, 2, 3, 5)$, the exact curve is

$$\Phi_\tau = \min\{16\tau, 2 + 8\tau, 16\}. \quad (19)$$

It changes at $\tau = 1/4$ and $\tau = 7/4$. The middle regime pairs the two inner levels, 2 and 3. The last regime instead pairs 0 with 2, and 3 with 5. Independent gap thresholding would miss this rearrangement.

5 Verification and scope

5.1 Numerical falsification checks

Both programs and both recorded receipts are published with this paper at <https://hypnosmath.org/research/antiunitary-symmetry-as-a-matching-problem>. The accompanying `verify.py` uses seed 20260928 and writes its full results, versions, and source hash to `verification.json`. The checks use generated finite inputs; they do not query the live Hypnos process or infer a theorem from zero data.

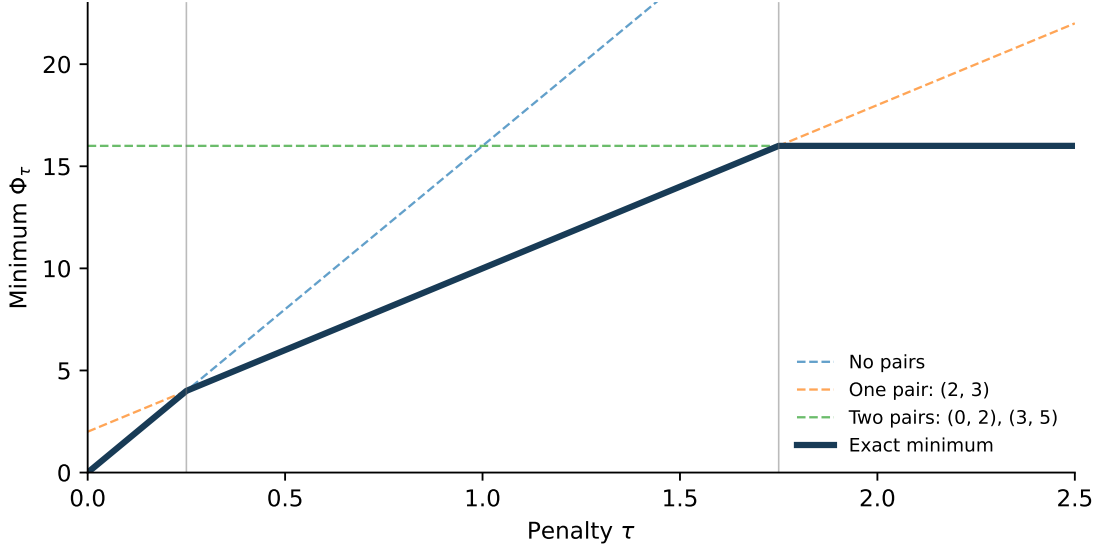


Figure 1: Exact penalty curve for $(0, 2, 3, 5)$. Dashed lines are the best costs with zero, one, or two pairs. The lower envelope is the unitary optimum. Increasing the penalty can replace an existing pair.

Check	Number of cases
Weighted matching versus exhaustive subset recurrence, $1 \leq n \leq 10$	1,000
Haar unitaries versus the analytic lower bound, $1 \leq n \leq 12$	4,800
Independent complex skew contractions, $2 \leq n \leq 9$	240
Scalar recurrence versus general matching, $1 \leq n \leq 30$	1,500
Direct continuous optimization from random initial points, $2 \leq n \leq 6$	120
Basis covariance and pointwise perturbation bounds, $2 \leq n \leq 8$	70
Exact rational objective checks, $1 \leq n \leq 7$	210

All exact matching comparisons agreed. Constructed optimizers agreed with their objective values to 3.6×10^{-15} . The energy decomposition's largest absolute discrepancy was 1.8×10^{-13} , and the odd-dimensional operator-norm identity agreed to 1.6×10^{-15} . No sampled unitary beat the theorem beyond roundoff. The odd-set inequalities were checked on every odd subset in the Haar samples with $n \leq 8$, and on every odd subset in the independent skew-contraction samples.

Direct Broyden–Fletcher–Goldfarb–Shanno (BFGS) optimization reached the exact minimum within 10^{-7} in 96 of 120 attempts. Its stopping flag reported success in only 80 attempts; those two counts measure different things. Some attempts ended above the global minimum, with maximum excess 48. All attempts are retained. This behavior illustrates why a local optimizer is not a certificate. Its analytic gradient was checked by finite differences, with absolute discrepancy 1.5×10^{-6} .

A separate standard-library program, `verify_exact.py`, uses exact rational arithmetic on Cayley transforms of real skew matrices. Its 210 objective checks verified the decomposition, lower bound, and attaining witness without rounding. It also checked 1,270 odd-set inequalities and both stability estimates in 177 cases with a unique optimal matching. Three tied cases were excluded from the positive-gap estimate; the remaining 30 cases were one-dimensional. This second check covers the real orthogonal subset, not all complex unitaries.

These are implementation and falsification checks. The theorem is proved by the arguments in Sections 2–4 using Edmonds’ theorem; it has not been formalized in a proof assistant or reviewed by a human mathematician.

5.2 Provenance

Hypnos [10] is a research harness built at the direction of David Ross. Its proposal models are Gemma 4 31B, a dense model in 4-bit activation-aware weight quantization (AWQ), `QuantTrio/gemma-4-31B-it-AWQ`; Gemma 4 26B, a mixture-of-experts model with 4 billion active parameters, served in AWQ from a quantization of `google/gemma-4-26B-A4B-it`; and Qwen3 32B, a dense model in 4-bit AWQ, `Qwen/Qwen3-32B-AWQ`. They run through vLLM on hardware operated by Ross and generate claims from a notebook of structured records. Claude Opus (Anthropic), accessed through a subscription, reviews samples under pre-registered rules; claims selected for execution include a computation that the harness runs. Proposals, verdicts and execution records are retained in the private repository and database. The site <https://hypnosmath.org> publishes reviewed explanations and selected computational records.

A read-only snapshot taken September 28, 2026, at 11:20 p.m. EDT records the motivating chain. Run-18 node 2649 reported the Frobenius minimum 2 at dimensions 3 and 5. Node 2676 identified the negative-square set with unitary skew-symmetric matrices but left comparison of the two residuals open; equation (6) settles that comparison by an exact identity. Node 2677 demonstrated the eigenbasis-conjugation construction numerically. Node 2698 retained a weaker determinant-based operator-norm bound; equation (11) gives the exact value. Node 2707 measured the symmetrization minimum 1 at dimensions 3 and 5. These numerical and speculative records are motivation, not premises of the proof. Their original text and status are preserved in the separate provenance snapshot.

The weighted reduction, the contraction-to-matching argument, and the stability statements were developed in this session from that chain. The manuscript uses a fixed snapshot of the harness record. The classical residual identities and parity facts are separated from the paper’s proposed contribution.

5.3 Limits of the conclusion

The exact spectral formula requires a finite commuting Hermitian tuple. For a noncommuting tuple, Proposition 3.5 gives a bound if a commuting approximant is supplied; it does not construct such an approximant. The result uses squared Frobenius costs and the stated nonnegative symmetric weights. It does not solve arbitrary unitary optimization or assert the same reduction for other penalties.

The optimized antiunitary may depend on the whole tuple. It is therefore a distance to an algebraic symmetry condition, not a test for a physical time-reversal operator specified in advance. For a single observable the value is a function of consecutive spectral gaps alone (Proposition 4.1), so it certifies spectral pairing, not the symmetry of any prescribed antiunitary. In particular, every finite Hermitian matrix admits a commuting antiunitary with square $+I$ by conjugating in an eigenbasis. This existence fact alone cannot identify a random-matrix ensemble or exclude a spectral realization. No claim about the Riemann hypothesis, zeta-zero simplicity, or infinite-dimensional operators follows here.

The related-work search made when the manuscript was written examined the primary texts of Edmonds, Loring and Higham and the bibliographic record of Wigner, and targeted combinations of antiunitary optimization, Kramers defects, skew-symmetric contractions, squared-entry maps,

and matching polytopes. It did not locate the penalty identity (3) in the sources examined. This supports presenting it as the contribution of this manuscript, while not establishing an exhaustive priority claim. Two independent reviews of the manuscript, conducted after this search, also failed to find the identity or the hull lemma in print; both found the adjacent work of Gil [7] on coherence with prescribed intrinsic populations, which the manuscript's first version did not cite, and older work on congruence orbits with prescribed singular values, which concerns selected entries of a single orbit rather than the squared-entry hull or the weighted identity. A further search on 2026-10-01, for work posted after those reviews, found one adjacent paper, Looi's stability theorem for Wigner symmetries [8], distinguished in Section 1, and nothing stating the identity. Kramers and Mirsky were cited after the reviews. The full texts of Wigner and Mirsky were not obtained by the search or by any review; that of Kramers was obtained in the fourth review; the reviews record which originals they reached. A search on 2026-10-02 covered the literature on unistochastic and orthostochastic matrices (the matrices of squared moduli of the entries of unitary and of orthogonal matrices) and found no statement of the identity (3) or of Lemma 2.2 in the sources examined. It found one further adjacent paper, posted 2026-09-29: Miyazaki, Kuroiwa and Murao [9] quantify the time-reversal violation of quantum channels relative to an antiunitary fixed in advance; here the antiunitary is the variable of the optimization and the observables are fixed.

References

- [1] J. Edmonds. Maximum matching and a polyhedron with 0,1-vertices. *Journal of Research of the National Bureau of Standards, Section B*, 69B(1-2):125–130, 1965. [doi:10.6028/jres.069B.013](https://doi.org/10.6028/jres.069B.013).
- [2] E. P. Wigner. Normal form of antiunitary operators. *Journal of Mathematical Physics*, 1(5):409–413, 1960. [doi:10.1063/1.1703672](https://doi.org/10.1063/1.1703672).
- [3] T. A. Loring. Transforming antiunitary symmetries to a normal form. *Expositiones Mathematicae*, 44(2):125737, 2026. [doi:10.1016/j.exmath.2025.125737](https://doi.org/10.1016/j.exmath.2025.125737). Cited from arXiv:2508.13004v2, <https://arxiv.org/abs/2508.13004>.
- [4] N. J. Higham. Matrix nearness problems and applications. In M. J. C. Gover and S. Barnett, editors, *Applications of Matrix Theory*, pages 1–27. Oxford University Press, 1989. <https://nhigham.com/wp-content/uploads/2023/10/high89n.pdf>.
- [5] H. A. Kramers. Théorie générale de la rotation paramagnétique dans les cristaux. *Proceedings of the Royal Netherlands Academy of Arts and Sciences*, 33:959–972, 1930. <https://dwc.knaw.nl/DL/publications/PU00015981.pdf>.
- [6] L. Mirsky. Symmetric gauge functions and unitarily invariant norms. *The Quarterly Journal of Mathematics*, 11(1):50–59, 1960. [doi:10.1093/qmath/11.1.50](https://doi.org/10.1093/qmath/11.1.50).
- [7] J. J. Gil. Entropic and geometric population–coherence complementarity in finite-dimensional quantum states. *Entropy*, 28(8):877, 2026. [doi:10.3390/e28080877](https://doi.org/10.3390/e28080877).
- [8] S. Looi. Stability of isometries of quantum states. arXiv:2609.37133, 2026. <https://arxiv.org/abs/2609.37133>.
- [9] J. Miyazaki, K. Kuroiwa and M. Murao. Dynamical resource theory of time-reversal symmetry breaking. arXiv:2609.36408, 2026. <https://arxiv.org/abs/2609.36408>.

- [10] Claude Fable 5, at the direction of D. Ross, Self-grading without self-deception: verdict discipline for a long-running mathematical research agent, manuscript (August 2026), Hypnos Math, <https://hypnosmath.org>.